Math 45 SSM 2/e 7.2 Multiplying and Dividing Rational Expressions (Day 2)

Today: 2) Dividing Rational Expressions.

Perform the indicated operations and simplify completely.

① $\frac{12}{25} \div \frac{24}{35}$

$= \frac{12}{25} \cdot \frac{35}{24}$

step 1: Write as multiply by reciprocal.

step 2: Factor completely.

$12 = 2 \cdot 2 \cdot 3$
$25 = 5 \cdot 5$
$35 = 5 \cdot 7$
$24 = 2 \cdot 2 \cdot 2 \cdot 3$

$= \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{3}}{\cancel{5} \cdot 5} \cdot \frac{\cancel{5} \cdot 7}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3}}$

step 3: Divide out or "cancel" common factors that appear in both the numerator and denominator.

{This may require the quotient rule exponent law.}

$= \boxed{\frac{5}{7}}$

② $\frac{20x^5}{3y} \div \frac{4x^2}{15y^5}$

$= \frac{20x^5}{3y} \cdot \frac{15y^5}{4x^2}$

$= \frac{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot x^5}{\cancel{3}\, y} \cdot \frac{\cancel{3} \cdot 5\, y^5}{\cancel{2} \cdot \cancel{2}\, x^2}$ cancel common factors

$= \frac{5 \cdot 5\, x^5\, y^5}{x^2\, y}$ align like bases

$= 25\, x^{5-2}\, y^{5-1}$ subtract exp

$= \boxed{25x^3y^4}$

③ $\frac{x+3}{2x-8} \div \frac{9}{4x}$

$= \frac{x+3}{2x-8} \cdot \frac{4x}{9}$ multiply by reciprocal

$= \frac{(x+3)}{2(x-4)} \cdot \frac{4x}{9}$ factor completely

$= \frac{4x(x+3)}{2 \cdot 9(x-4)}$ reduce $\frac{4}{2}=2$

$= \boxed{\frac{2x(x+3)}{9(x-4)}}$

④ $\frac{y^2-9}{2y^2-y-15} \div \frac{3y^2+10y+3}{2y^2+y-10}$

$= \frac{y^2-9}{2y^2-y-15} \cdot \frac{2y^2+y-10}{3y^2+10y+3}$ multiply by reciprocal

$= \frac{\cancel{(y-3)}\cancel{(y+3)}}{\cancel{(y-3)}\cancel{(2y+5)}} \cdot \frac{\cancel{(2y+5)}(y-2)}{\cancel{(y+3)}(3y+1)}$

$= \boxed{\frac{y-2}{3y-1}}$

factor completely

1) $y^2-9 = (y-3)(y+3)$
difference of squares

2) $2y^2-y-15$ -30, -6, $+5$, -1

$= 2y^2-6y+5y-15$
$= 2y(y-3)+5(y-3)$
$= (y-3)(2y+5)$

3) $2y^2+y-10$ -20, 5, -4, 1
$= 2y^2+5y-4y-10$
$= y(2y+5)-2(2y+5)$
$= (2y+5)(y-2)$

4) $3y^2+10y+3$ 9, 9, 1, 10
$= 3y^2+9y+y+3$
$= 3y(y+3)+1(y+3)$
$= (y+3)(3y+1)$

⑤ $\dfrac{a^2+3a+2}{a^2-9} \div (a+2)$

$= \dfrac{a^2+3a+2}{a^2-9} \cdot \dfrac{1}{(a+2)}$ multiply by reciprocal

factor completely

1) $a^2+3a+2 = (a+2)(a+1)$
2) $a^2-9 = (a-3)(a+3)$
3) 1
4) $(a+2)$

$= \dfrac{\cancel{(a+2)}(a+1)}{(a+3)(a-3)} \cdot \dfrac{1}{\cancel{(a+2)}}$

$= \boxed{\dfrac{(a+1)}{(a+3)(a-3)}}$

Recall: Leave fractions factored.

⑥ $\dfrac{\dfrac{12x}{5x+20}}{\dfrac{4x^2}{x^2-16}}$ ← This division bar becomes ÷ symbol.

$= \dfrac{\left(\dfrac{12x}{5x+20}\right)}{\left(\dfrac{4x^2}{x^2-16}\right)}$

$= \dfrac{12x}{5x+20} \div \dfrac{4x^2}{x^2-16}$ write with ÷ symbol

$= \dfrac{12x}{5x+20} \cdot \dfrac{x^2-16}{4x^2}$ multiply by reciprocal

$= \dfrac{12 \cdot x}{5\cancel{(x+4)}} \cdot \dfrac{(x-4)\cancel{(x+4)}}{4x^2}$ factor completely

$= \dfrac{12 \cdot x \cdot (x-4)}{4 \cdot 5 \cdot x^2}$ reduce $\frac{12}{4} = 3$, $\frac{x}{x^2} = \frac{1}{x}$

$= \boxed{\dfrac{3(x-4)}{5x}}$ leave result factored

⑦ $$\frac{\frac{x^2+25}{x^7}}{\frac{x^2+5x}{x^2}}$$

$= \frac{x^2+25}{x^7} \div \frac{x^2+5x}{x^2}$ rewrite as division

$= \frac{(x^2+25)}{x^7} \cdot \frac{x^2}{x^2+5x}$ rewrite as multiply by reciprocal of 2nd fraction

$= \frac{(x^2+25)}{x^7} \cdot \frac{x^2}{x(x+5)}$ sum of squares x^2+25 is prime; factor GCF x from x^2+5x

$= \frac{x^2}{x^8} \cdot \frac{(x^2+25)}{(x+5)}$ rewrite like bases near each other

$= \boxed{\frac{(x^2+25)}{x^6(x+5)}}$ exponent law

⑧ $$\frac{x^2-5x}{x^3+125} \div \frac{x-5}{x^2-5x+25}$$

$= \frac{x^2-5x}{x^3+125} \cdot \frac{x^2-5x+25}{x-5}$ rewrite as multiply by reciprocal of 2nd fraction

$= \frac{x(x-5)}{(x+5)(x^2-5x+25)} \cdot \frac{(x^2-5x+25)}{(x-5)}$ factor GCF x from x^2-5x; factor sum of cubes x^3+125; add parentheses to prime polynomials $(x^2-5x+25)$ and $(x-5)$

$= \frac{x\,(x-5)(x^2-5x+25)}{(x-5)(x^2-5x+25)(x+5)}$ write like bases near each other & cancel common factors

$= \boxed{\frac{x}{x+5}}$

Extras

(9) $\frac{5a-5b}{70a^2} \div \frac{10b-10a}{21c}$

(10) $\frac{a^2+2a-8}{2a^2+a-3} \div \frac{a^2+4a}{2a^2-a-6}$

(11) $\frac{y^2-2y-8}{y^2+y} \div (y+2)$

(12) $\frac{2a-8+a^2}{3-a-2a^2} \div \frac{4a+a^2}{6+a-2a^2}$

(13) $\frac{y^2-2y-8}{y^2+y} \div \frac{y-2}{4-y^2}$

(14) $\dfrac{\frac{16z}{7z+14}}{\frac{8z^2}{z^2-4}}$

(15) $\dfrac{\frac{3x-3y}{5z^2}}{\frac{6y-6x}{20z}}$

7.2-17 **Perform the indicated operation and simplify.**

$$-\frac{5x^3}{6x} \div \frac{10x^2}{9x^2}$$

- A. $\frac{3x^2}{4}$
- B. $-\frac{3x^2}{2}$
- C. $-\frac{x^2}{4}$
- ✔ D. $-\frac{3x^2}{4}$

rewrite as multiply by reciprocal

$$= -\frac{5x^3}{6x} \cdot \frac{9x^2}{10x^2}$$

reduce numbers $-\frac{\cancel{5}^{1}}{6} \cdot \frac{9}{\cancel{10}_{2}} = -\frac{1}{\cancel{6}_{2}} \cdot \frac{\cancel{9}^{3}}{2} = -\frac{3}{4}$

simplify x's:

option 1: add exp across, then subtract

$$\frac{x^5}{x} \cdot \frac{x^2}{x^2} = \frac{x^7}{x^3} = x^4$$

option 2: subtract exp

$$\frac{x^5}{x} \cdot \frac{x^2}{x^2} = x^4 \cdot 1 = x^4$$

$$= \boxed{-\frac{3x^4}{4}}$$

7.2.43 Perform the division.

NEXT

$$\frac{(g-9)^2}{9-g^2} \div \frac{g^2-81}{g-3}$$

$$\frac{(g-9)^2}{9-g^2} \div \frac{g^2-81}{g-3} = -\frac{g-9}{(g+3)(g+9)}$$

YOU ANSWERED: $-\frac{g-3}{(3+g)(3+g)}$

rewrite as multiply by reciprocal

$$\frac{(g-9)^2}{9-g^2} \cdot \frac{g-3}{g^2-81}$$

standard form

$$\frac{(g-9)^2}{-g^2+9} \cdot \frac{g-3}{g^2-81}$$

(A) (B)

factor completely

$$\frac{(g-9)(g-9)}{-(g-3)(g+3)} \cdot \frac{(g-3)}{(g-9)(g+9)}$$

(A) $-g^2+9$

GCF $= -(g^2-9)$

diff sq $= -(g-3)(g+3)$

(B) g^2-81

diff sq $= (g-9)(g+9)$

cancel common factors.

$$\frac{g-9}{g-9}, \frac{g-3}{g-3}$$

$$= \boxed{-\frac{(g-9)}{(g+3)(g+9)}}$$

7.2.47 **Perform the indicated operation and simplify.**

$$\frac{\frac{4c-12}{80}}{\frac{3-c}{4}}$$

$$\frac{\frac{4c-12}{80}}{\frac{3-c}{4}} = -\frac{1}{5}$$

Rewrite with ÷ symbol

$$= \frac{4c-12}{80} \div \frac{3-c}{4}$$

Multiply by reciprocal

$$= \frac{4c-12}{80} \cdot \frac{4}{3-c}$$

Standard form

$$= \frac{4c-12}{80} \cdot \frac{4}{-c+3}$$

Factor completely

$$= \frac{4(c-3)}{80} \cdot \frac{4}{-1(c-3)}$$

Cancel common factors

$$= \boxed{\frac{-1}{5}}$$

7.2.51 Perform the division.

NEXT

$$\frac{y^2v}{8y^2-65yv+8v^2} \div \frac{(8yv^2)^2}{8y^2v-yv^2}$$

$$\frac{y^2v}{8y^2-65yv+8v^2} \div \frac{(8yv^2)^2}{8y^2v-yv^2} = \frac{y}{64v^2(y-8v)}$$ (Simplify your answer.)

Rewrite as multiply by reciprocal

$$\text{(A)}\ \frac{y^2v}{8y^2-65yv+8v^2} \cdot \frac{8y^2v-yv^2}{(8yv^2)^2}\ \text{(B)}\ \text{(C)}$$

Factor completely

$$= \frac{y^2v}{(8y-v)(y-8v)} \cdot \frac{yv(8y-v)}{64y^2v^4}$$

Cancel common factors:

$$\frac{(8y-v)}{(8y-v)},\ \frac{y^2}{y^2},\ \frac{v^2}{v^2}$$

$$= \boxed{\frac{y}{64v^2(y-8v)}}$$

(A) $8y^2-65yv+8v^2$

$(8y-v)(y-8v)$

OR guess & check

64, -64, -1, -64

$8y^2-64yv-yv+8v^2$

$8y(y-8v)-v(y-8v)$

$(8y-v)(y-8v)$

(B) $8y^2v-yv^2$

$=yv(8y-v)$

(C) $(8yv^2)^2$

$= 8^2 \cdot y^2 \cdot (v^2)^2$

$= 64y^2v^4$

7.2.57 **Perform the division.**

$$\frac{(a+2)^2}{a^2-4} \div \frac{-a^2+6a+16}{a^2-10a+16}$$

$$\frac{(a+2)^2}{a^2-4} \div \frac{-a^2+6a+16}{a^2-10a+16} = \square$$

write as multiply by reciprocal:

$$\frac{(a+2)^2}{a^2-4} \cdot \frac{a^2-10a+16}{-a^2+6a+16}$$

(A) (B) (C)

standard form ✓

factor completely:

$$\frac{(a+2)(a+2)}{(a+2)(a-2)} \cdot \frac{(a-8)(a-2)}{-(a-8)(a+2)}$$

Cancel common factors

$$\frac{a+2}{a+2}, \frac{a+2}{a+2}, \frac{a-2}{a-2}, \frac{a-8}{a-8}$$

The only thing left is $\boxed{-1}$

(A) a^2-4
diff of sq
$= (a-2)(a+2)$

(B) $a^2-10a+16$

16
-8 × -2
-10

$= (a-8)(a-2)$

(C) $-a^2+6a+16$
GCF -1

$= -(a^2-6a-16)$

-16
-8 × +2
-6

$= -(a-8)(a+2)$

7.2.63 **Perform the division.**

$$\frac{2s^3-2}{s^4-1} \div \frac{7s^2+7s+7}{s^3+s^2+s+1}$$

$$\frac{2s^3-2}{s^4-1} \div \frac{7s^2+7s+7}{s^3+s^2+s+1} = \frac{2}{7}$$

rewrite as multiply by reciprocal

Ⓐ $\frac{2s^3-2}{s^4-1} \cdot \frac{s^3+s^2+s+1}{7s^2+7s+7}$ Ⓒ

Ⓑ Ⓓ

Standard form ✓

factor completely

$$\frac{2(s-1)(s^2+s+1)}{(s^2+1)(s-1)(s+1)} \cdot \frac{(s^2+1)(s+1)}{7(s+1)(s+1)}$$

cancel common factors

$$\frac{(s-1)}{(s-1)}, \frac{(s+1)}{(s+1)}, \frac{(s^2+1)}{(s^2+1)}$$

$$= \boxed{\frac{2(s^2+s+1)}{7(s+1)^2}}$$

Ⓐ $2s^3-2$

GCF $= 2(s^3-1)$

diff cubes $= 2(s-1)(s^2+s+1)$

Ⓑ s^4-1

diff sq $= (s^2+1)(s^2-1)$

diff sq $= (s^2+1)(s-1)(s+1)$

Ⓒ s^3+s^2+s+1

grouping

$= s^2(s+1)+1(s+1)$

$= (s^2+1)(s+1)$

↑ sum sq prime

Ⓓ $7s^2+7s+1$

GCF

$= 7(s^2+s+1)$

perfect sq tri.

$= 7(s+1)^2$

7.2.71 Find $\frac{10x+10y}{x-y}$ squared divided by $100x^2-100y^2$.

$$\left(\frac{10x+10y}{x-y}\right)^2 \div (100x^2-100y^2) = \frac{x+y}{(x-y)^3}$$

(Type your answer in factored form.)

rewrite as multiply, not words

$$\left(\frac{10x+10y}{x-y}\right)^2 \cdot \frac{1}{100x^2-100y^2}$$

Order of operations $\Rightarrow$ ( ).

Ⓐ $\frac{10x+10y}{x-y} \cdot \frac{10x+10y}{x-y}$ · $\frac{1}{100x^2-100y^2}$ Ⓑ

Standard form ✓

Factor completely

$$= \frac{10(x+y)}{(x-y)} \cdot \frac{10(x+y)}{(x-y)} \cdot \frac{1}{100(x+y)(x-y)}$$

Ⓐ $10x+10y$
GCF $10(x+y)$

Ⓑ $100x^2-100y^2$
GCF
$100(x^2-y^2)$
diff of sq
$100(x-y)(x+y)$

Cancel common factors

$\frac{10 \cdot 10}{100}$, $\frac{x+y}{x+y}$

$$= \boxed{\frac{(x+y)}{(x-y)^3}}$$